

# 2022 现代分析与几何学术研讨会

## 会议手册



中国科学技术大学数学科学学院  
山东大学数学与交叉科学研究中心  
北京邮电大学理学院  
教育部非线性期望前沿科学中心

2022 年 8 月

尊敬的与会嘉宾:

欢迎您参加 2022 年 8 月 9-11 日举办的“2022 现代分析与几何”学术研讨会。本次会议由中国科学技术大学、山东大学和北京邮电大学联合举办,并得到科技部重点研发计划青年科学家项目《拟共形分析与几何》(2021YFA1002200)资助。

### 特邀报告人 (按姓氏字母排序)

曹道民 (中国科学院)	陈小杨 (同济大学)
戴蔚 (北京航空航天大学)	胡锡俊 (山东大学)
黄曼子 (湖南师范大学)	李文娟 (西北工业大学)
刘劲松 (中国科学院)	刘兆理 (首都师范大学)
乔建永 (北京邮电大学)	沈玉良 (苏州大学)
夏超 (厦门大学)	向长林 (三峡大学)
熊革 (同济大学)	熊金钢 (北京师范大学)
张会春 (中山大学)	张志涛 (中国科学院)
郑高峰 (华中师范大学)	周渊 (北京师范大学)

### 会议组织委员会 (按姓氏字母排序)

郭常予、韩邦先、王宏钰

### 会议学术委员会 (按姓氏字母排序)

李嘉禹、刘聪文、麻希南、任广斌、张希

## 8月9日会议日程

腾讯会议地址：669-7346-0934, 会议密码：1958

日期	时间	报告人	题目	主持人	
8月9日	8:30-8:50		开幕式		
	8:50-9:40	曹道民	Helical symmetry solutions for 3D incompressible Euler equations in an infinite cylinder	麻希南	
	9:50-10:40	刘兆理	Quasilinear Schrödinger equations involving singular potentials		
	10:50-11:40	张志涛	Dynamics of nonlinear hyperbolic equations of Kirchhoff type		
	午 休				
	14:00-14:50	张会春	The Regularity for free boundary problems on non-smooth metric measure spaces	李嘉禹	
	15:00-15:50	熊金钢	Harmonic maps with finite hyperbolic distances to the Extreme Kerr		
	16:00-16:50	夏超	Heintze-Karcher's inequality and Alexandrov's soap bubble theorem		

8 月 10 日会议日程

腾讯会议地址：669-7346-0934, 会议密码：1958

日期	时间	报告人	题目	主持人	
10 日	8:50-9:40	刘劲松	Boundary regularity of isometries between infinitely flat complex domains	任广斌	
	9:50-10:40	沈玉良	Weil-Petersson Teichmüller 空间及其应用和推广		
	10:50-11:40	黄曼子	Gromov hyperbolicity and Uniformity		
	午 休				
	14:00-14:50	乔建永	关于复动力系统中 Julia 集面积的研究	张希	
	15:00-15:50	熊革	Recursion formulas, concentration polytopes, and sharp affine isoperimetric inequalities for volume decomposition functionals		
	16:00-16:50	陈小杨	New Bochner type theorems		

## 8 月 11 日会议日程

腾讯会议地址: 669-7346-0934, 会议密码: 1958

日期	时间	报告人	题目	主持人	
11 日	8:50-9:40	胡锡俊	Index theory and stability of elliptic relative equilibria in Planar n-body problem	李工宝	
	9:50-10:40	郑高峰	$L^p$ regularity theory for even order elliptic systems with antisymmetric first order potentials		
	10:50-11:40	向长林	Riviere's conjecture and conservation law of harmonic mappings		
	午 休				
	14:00-14:50	周渊	Existence of hyperbolic motions to a class of Hamiltonians and generalized $N$ -body system via a geometric approach	蒋仁进	
	15:00-15:50	戴蔚	Uniform a priori estimates for critical order Lane-Emden system in arbitrary dimensions		
	16:00-16:50	李文娟	Sharp convergence for sequences of Schrödinger means and related generalizations		
	离开会议				

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报告摘要信息

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曹道民（中国科学院）

报告题目：Helical symmetry solutions for 3D incompressible Euler equations in an infinite cylinder

摘要：In this talk we are interested in solutions whose vorticities are large and concentrated uniformly near a smooth curve  $\gamma(t)$  embedded in entire  $R^3$ . This type of solutions, vortex filaments, are classical objects of fluid dynamics. Under suitable assumptions it is known to some extent that the curve evolves by its binormal flow. Two special kinds of binormal flows are traveling circle and rotating-translating helix. Solutions concentrating near a traveling circle is called vortex ring which have been studied extensively. In this talk, we will present existence of solutions near rotating-translating helix. The general case is called vortex filament conjecture which is still a well-known open problem. This talk is based on a joint paper with Wan Jie at Beijing University of Technology.

陈小杨（同济大学）

报告题目：New Bochner type theorems

摘要：A classical theorem of Bochner asserts that the isometry group of a compact Riemannian manifold with negative Ricci curvature is finite. In this talk we discuss several extensions of Bochner's theorem by allowing "small" positive Ricci curvature. This is a joint work with Fei Han.

戴蔚（北京航空航天大学）

报告题目: Uniform a priori estimates for critical order Lane-Emden system in arbitrary dimensions

摘要: In this talk, we establish uniform a priori estimates for positive solutions to the  $n$ -th order superlinear Lane-Emden system in bounded domains with Navier boundary conditions in  $\mathbb{R}^n$  ( $n \geq 3$ ). This is joint work with Leyun Wu.

胡锡俊 (山东大学)

报告题目: Index theory and stability of elliptic relative equilibria in Planar  $n$ -body problem

摘要: It is well known that a planar central configuration of the  $n$ -body problem gives rise to solutions where each particle moves on a specific Keplerian orbit while the totality of the particles move on a homographic motion. Following Meyer and Schmidt, we call such solutions elliptic relative equilibria. Some famous examples such as Lagrangian orbits, Euler orbits, etc. have important background in the solar system. In this talk, we will introduce several new methods to study its stability. Based on joint works with Yiming Long, Shanzhong Sun and Yuwei Ou.

黄曼子 (湖南师范大学)

报告题目: Gromov hyperbolicity and Uniformity

摘要: In this talk, we discuss the geometric properties of Gromov hyperbolic domains. The first property is the Gromov hyperbolicity. The second is the cigar condition of the quasihyperbolic geodesic in John domain. The third is the Gehring-Hayman property of the quasihyperbolic geodesic in inner uniform. At last, we get that every Gromov hyperbolic John domain in Banach spaces has inner uniformity.

李文娟 (西北工业大学)

报告题目: Sharp convergence for sequences of Schrödinger means and related generalizations

摘要: For decreasing sequences  $\{t_n\}_{n=1}^{\infty}$  converging to zero, we obtain the almost everywhere convergence results for sequences of Schrödinger means  $e^{it_n\Delta}f$ , where  $f \in H^s(\mathbb{R}^N)$ ,  $N \geq 2$ . The convergence results are sharp up to the endpoints, and the method can also be applied to get the convergence results for the fractional Schrödinger means and nonelliptic Schrödinger means. This is a joint work with Dr. Huiju Wang and Prof. Dunyan Yan.

刘劲松 (中国科学院)

报告题目: Boundary regularity of isometries between infinitely flat complex domains

摘要: In this talk, by using the Gehring-Hayman-type Theorem on some complex domains, we prove that the isometries (with respect to Kobayashi metrics) between certain domains with infinitely flat boundary points extend continuously to the boundaries. Furthermore, some regularity results of boundary extension maps are given.

刘兆理 (首都师范大学)

报告题目: Quasilinear Schrödinger equations involving singular potentials

摘要: In this talk, I shall talk about existence of multiple solutions of the quasilinear Schrödinger equation

$$-\Delta u + V(x)u + \frac{\kappa}{2}\Delta(u^2)u = h(u), \quad u \in H^1(\mathbb{R}^N),$$

where  $N \geq 3$ ,  $\kappa$  is a real parameter,  $V(x) = V(|x|)$  is a potential allowed to be singular at the origin and  $h: \mathbb{R} \rightarrow \mathbb{R}$  is a nonlinearity satisfying conditions similar to those in the paper [Arch. Rational Mech. Anal., 82 (1983), 347-375] by H. Berestycki and P.-L. Lions. We establish the existence of infinitely many radial solutions for  $\kappa < 0$  and the existence of more and more radial solutions as  $\kappa \downarrow 0$ . This is joint work with Yongtao Jing and Haidong Liu (靖永涛和刘海东).



乔建永（北京邮电大学）

报告题目：关于复动力系统中 Julia 集面积的研究

摘要：在介绍复动力系统基本理论和研究问题的基础上，回顾混沌集合--Julia 集面积问题的研究；系统总结面积问题的研究结果和 Douady's Plan；解读 Buff 和 Cheritat 的著名工作，以及 BC-猜测；最后介绍我们关于 BC-猜测的证明。

沈玉良（苏州大学）

报告题目：Weil-Petersson Teichmüller 空间及其应用和推广

摘要：介绍 Weil-Petersson Teichmüller 空间的研究背景和在分析方面的一些研究进展和推广，以及在 $H^{3/2}$ 向量场理论中的若干应用。

夏超（厦门大学）

报告题目：Heintze-Karcher's inequality and Alexandrov's soap bubble theorem

摘要：Heintze-Karcher's inequality is an interesting geometric inequality for embedded closed hypersurfaces, which can be used to prove Alexandrov's soap bubble theorem on embedded closed CMC hypersurfaces in the Euclidean space.

In this talk, we introduce two extensions, one is on closed hypersurfaces in warped product manifolds, the other is on capillary hypersurfaces in the half-space, a ball or a wedge. The main focus is on two different approaches towards the Heintze-Karcher inequality. This talk is based on separate joint works with Junfang Li and with Xiaohan Jia, Guofang Wang and Xuwen Zhang.

向长林（三峡大学）

报告题目：Riviere's conjecture and conservation law of harmonic mappings

摘要：In this talk I will discuss Riviere's regularity conjecture on weakly harmonic mappings and related progress made by Naber and Valtota on minimizing and stationary harmonic mappings via their quantitative stratification theory. Naber and Valtota's approach depends heavily on monotonicity formula of stationary harmonic mappings, and thus seems impossible to deal with general weakly harmonic mappings. We explored a different method: the method of conservation law. This method is inspired by the study of 2 dimensional harmonic mappings, but still have severe difficulty to get progress.

熊革（同济大学）

报告题目：Recursion formulas, concentration polytopes, and sharp affine isoperimetric inequalities for volume decomposition functionals

摘要：New sharp affine isoperimetric inequalities for the volume decomposition functionals are established. To attack these extremal problems, we find the recursion formulas of volume decomposition functionals, then introduce concentration polytope and characterize its geometric structure, especially its vertices and 1-dimensional faces. The concentration polytope intuitively reinterprets the discrete logarithmic Minkowski problem.

熊金钢（北京师范大学）

报告题目：Harmonic maps with finite hyperbolic distances to the Extreme Kerr

摘要：We study harmonic maps with finite hyperbolic distances to the Extreme Kerr from domains in the 3d Euclidean space to the hyperbolic plane. We prove that such maps have unique tangent maps at the black hole horizon. This particularly completes the regularity problem of harmonic maps arising from stationary axi-symmetric solutions of the Einstein vacuum field equations with multiple black holes, dating back to Weinstein 1989 and Li-Tian 1992. This is joint with Q. Han, M. Khuri and G. Weinstein.

张会春（中山大学）

报告题目：The Regularity for free boundary problems on non-smooth metric measure spaces.

摘要：In this talk, we will introduce some regularity results for a Bernoulli-type one-phase free boundary problem on metric measure spaces with a generalized lower Ricci bound, the so-called Riemannian curvature-dimension condition  $RCD(K, N)$ . The free boundary problem is similar as the minimal surfaces and harmonic maps. We consider the Lipschitz regularity of solutions, the partial regularity of the free boundary, and the estimate of size of singular set in the free boundary. This is a joint work with Chung-Kwong Chan, and Xi-Ping Zhu.

张志涛（中国科学院，江苏大

报告题目：Dynamics of nonlinear hyperbolic equations of Kirchhoff type

摘要：We study the initial boundary value problem of the important hyperbolic Kirchhoff equation

$$u_{tt} - (a \int_{\Omega} |\nabla u|^2 dx + b)\Delta u = \lambda u + |u|^{p-1}u, \quad u(t, x)|_{\partial\Omega} = 0,$$

where  $a, b > 0, p > 1, \lambda \in \mathbb{R}$  and the initial energy is arbitrarily large. We prove several new theorems on the dynamics such as the boundedness or finite time blow-up of solution under the different range of  $a, b, \lambda$  and the initial data.

郑高峰（华中师范大学）

报告题目： $L^p$  regularity theory for even order elliptic systems with antisymmetric first order potentials

摘要：In this talk, we are concerned with the optimal interior regularity theory to the system

$$\Delta^m u = \sum_{l=0}^{m-1} \Delta^l \langle V_l, du \rangle + \sum_{l=0}^{m-2} \Delta^l \delta(w_l du) + f \quad \text{in } B^{2m}$$

Combining the conservation law established by Longueville-Gastel for homogeneous system and some new ideas together, we obtain optimal Hölder continuity and sharp  $L^p$  regularity theory, similar to that of Sharp and Topping in two order case, for weak solutions to the above

inhomogeneous system. Our results can be applied to study heat flow and bubbling analysis for polyharmonic mappings. This is a joint work with Prof. Chang-Yu Guo and Prof. Chang-Lin Xiang.

周渊（北京师范大学）

报告题目：Existence of hyperbolic motions to a class of Hamiltonians and generalized N-body system via a geometric approach

摘要：For the classical N-body problem in  $R^d$  with  $d \geq 2$ , Maderna-Venturelli in their remarkable paper [Ann.Math. 2020] proved the existence of hyperbolic motions based on some PDE approach. We give a geometric proof for such existence. Moreover, our geometric approach works for general Hamiltonians  $\frac{1}{2}\|p\|^2 - F(x)$ , where  $F(x) \geq 0$  is lower semicontinuous and decreases very slowly to 0 faraway from collisions  $\Delta$ , and also works for the corresponding generalized N-body system  $\ddot{x} = \nabla_x F(x)$  when  $F(x) \in C^2(R^{Nd} \setminus \Delta)$  in addition.